IDENTIFICATION METHODS OF BOUNDARY CONDITIONS FOUND FROM THE SOLUTION OF THE INVERSE HEAT-CONDUCTION PROBLEM

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We investigate the identification methods of equivalent boundary conditions for a system of bodies exchanging heat. A system of equations is presented which has been constructed from the experimental data for the heat exchange in vacuum.

The solution of the inverse heat-conduction problem as an analysis of the temperature data allows for any body a number of equivalent boundary conditions. It is important in applications to determine the actual conditions of the external heat exchange: the components of the external heat fluxes, the degree of blackness, the coefficients of accommodation and temperature of the surfaces, the heat-transfer coefficients, angle coefficients, etc. This is the problem of their identification. The heat fluxes q_k, $k = 1, 2, \ldots, l$, flowing through the boundary surfaces of the bodies, are assumed to be known from a previous solution of the inverse heat-conduction problem. Each of the heat fluxes consists of a sum of heat fluxes q_k, $j = 1, 2, \ldots, n_k$, which depends on $m_{k,j,i}$, $i = 1, 2, \ldots, m_{k,j}$, and on the parameters $\mu_{k,j}$, i.e.

$$q_K = \sum_{j=1}^{n_K} q_{k,j}(\mu_{k,j,i}). \tag{1}$$

The most natural way of dividing the heat fluxes q_k into components $q_{k,j}$ is their construction from experimental data, and by solving a sufficiently large number of equations connecting the quantities q_k with the variable parameters $\mu_{k,j,i}$. To solve these problems we have to specify the functional dependences $q_{k,j}(\mu_{k,j,i})$.

For all typical kinds of heat exchange we can write down these functional dependences. They are the Fourier equation, Stefan–Boltzmann equation, the conditional dependences for the convective heat exchange, etc. The parameters $\mu_{k,j,i}$ which appear in these equations are the coefficients of thermal conductivity, heat transfer, and activity, the degree of blackness, etc. The total number of unknowns is equal to the number of unknown parameters of interest on all surfaces of the problem. The formulation and solution of these equations, together with the methods of obtaining the input data, all belong to the problem of boundary-condition identification.

The identification methods can be divided into:

1) those using the direct measurement of the heat-flux components and of the parameters determining the boundary conditions;

2) those using the variation of the state variables of the system: the pressure of the medium and the temperature of the surfaces participating in the heat exchange;

3) those based on the introduction of external perturbations of the reflection coefficients, accomodation coefficients, and other properties of the boundary surfaces of the bodies, and also the perturbations of the specific heat, thermal conductivity, and other properties of the bodies of the system;

4) combined methods.

Many variations are possible in the organization of the experiments based on any of the possible identification methods.

Only a limited number of parameters or heat-flux components can be directly determined in industrial conditions or even by using sophisticated instruments. The present identification method can therefore be only

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used together with other methods (excluding the simpler experiments). It can be conveniently used to check the results of identification by other methods.

The method using the variation of the state parameters of the system is applicable for any number of unknown parameters. The only limitations imposed on it are by the limitations of the numerical technique or by the length of time needed to process the experimental results.

In the formulation of the equations it is necessary to use experimental data (together with the solution of the inverse heat-conduction problem) on the total heat fluxes and on the variable state parameters.

As an example we shall investigate the system of l bodies enclosed in a vacuum chamber. The heat exchange between the bodies is due to radiative heat transfer and to thermal conductivity of the residual gas. At every instant of time the heat exchange between the bodies is described by l equations of the form

$$Q_k = \frac{PF_k\beta}{\sqrt{B}} \sum_{\omega=1}^{l} \alpha_\omega T_\omega^{\frac{1}{2}} \varphi_{\omega,k} - \frac{PF_k\beta}{\sqrt{B}} \alpha_k T_k^{\frac{1}{2}} \sum_{\omega=1}^{l} \varphi_{\omega,k}$$
$$+ \sigma F_k \sum_{\omega=1}^{l} \frac{\varepsilon_\omega}{A_\omega} \frac{T_\omega^4 \varphi_{k,\omega}}{1 + \varphi_{k,\omega}\left(\frac{1}{A_\omega} - 1\right) + \varphi_{\omega,k}\left(\frac{1}{A_k} - 1\right)}$$
$$- \sigma F_k T_k^4 \frac{\varepsilon_k}{A_k} \sum_{\omega=1}^{l} \frac{\varphi_{k,\omega}}{1 + \varphi_{k,\omega}\left(\frac{1}{A_\omega} - 1\right) + \varphi_{\omega,k}\left(\frac{1}{A_k} - 1\right)}. \quad (2)$$

The first two sums in these equations describe the heat exchange by thermal conductivity of the residual gas and the other two by radiation.

If the unknowns in Eqs. (2) are, for example, the accomodation and reflection coefficients, and the degree of blackness, the total number of unknowns is $3l$. For their determination we construct $3l$ equations of the type (2) in which we change either T_ω or P, and use the experimentally determined (with the use of the solution of the inverse heat-conduction problem) quantities Q_k. Since the equations are constructed for l bodies the number of experiments used to construct these equations must be such that, for example, the temperature of each surface is changed not less than three times (given by the number of parameters we want to determine for the surfaces). If it is the temperatures of all surfaces that are independently varied in each experiment, the required number of experiments will be the minimum three. If the number of investigated parameters is increased (for example, by the addition of the coefficients of mutual irradiance) the number of required experiments is increased accordingly.

An important application of the present method is to describe all experiments using in the analysis the same functional dependences $q_{k,j}(\mu_{k,j,i})$. This represents the necessity of self-similarity conservation with respect to the variable parameters in all experiments. The conditions for self-similarity are in our example:

1) free molecular motion of the gas which is given by the condition $kn \ll 1$;

2) negligibly small changes of spectral properties in the whole range of wavelengths of the radiative emission when the temperature of the bodies is changed.

If the experiments are set up so that the pressure P is changed, it follows from the expression (2) that the radiative transfer parameters cannot be found from the experimental data. On the other hand, if the pressure does not change in the experiments we cannot find the accommodation coefficients of the surfaces.

Increasing the number of experiments above the number required for the determining the investigated parameters can be used to increase the accuracy of the determination of these parameters by well known methods.

It is possible to elaborate the method of boundary-condition identification to use nonstationary thermal processes, provided they are quasistationary. This simplifies the experiments by reducing the technological requirements needed for the processing of the results.

As an example of the application of the method of thermal constants perturbation we shall investigate the following two-stage method of the boundary-condition identification. In the first stage we shall determine the effective values of the specific heat C_{ef} at any point of the bodies by varying the heat evolution in the bodies of the system, or by varying the amount of heat passing through their boundary surfaces (using the solution of the inverse heat-conduction problem). To this aim we consider the thermal balance at chosen points of the temperature measurement at two instances of time τ:

$$C_{ef}\frac{dT}{d\tau}=\sum_{\omega=1}^{l}Q_{\omega}+Q_{b}.$$

The quantities C_{ef} are now found from the expression

$$C_{ef}=\frac{\left(\sum_{\omega=1}^{m}Q_{\omega}+Q_{b}\right)_1-\left(\sum_{\omega=1}^{m}Q_{\omega}+Q_{b}\right)_2}{\left(\frac{dT}{d\tau}\right)_1-\left(\frac{dT}{d\tau}\right)_2},$$

where the subscripts 1 and 2 refer to the two instants of time.

In the second stage we use C_{ef} and apply arbitrary perturbations of the thermal constants of the bodies or their boundary surfaces to determine the unperturbed values of the thermal constants from the temperature readings at various points:

$$C_{ef}\left[\left(\frac{dT}{d\tau}\right)_1-\left(\frac{dT}{d\tau}\right)_2\right]=(LT)_1-(LT)_2.$$

The known parameters are in this case the perturbed values of the thermal constants or the magnitudes of the perturbations.

In the previous example, the operator LT for each boundary surface (of index k) has the following form:

a) If the perturbations are used to obtain the degree of blackness ε, the reflection coefficient A, or the ratio ε/A,

$$(LT)_k=\sigma F_k\sum_{\omega=1}^{l}\frac{\varepsilon_{\omega}}{A_{\omega}}\frac{T_{\omega}^4\varphi_{k,\omega}}{1+\varphi_{k,\omega}\left(\frac{1}{A_{\omega}}-1\right)+\varphi_{\omega,k}\left(\frac{1}{A_k}-1\right)}$$
$$-\sigma F_kT_k^4\frac{\varepsilon_k}{A_k}\sum_{\omega=1}^{l}\frac{\varphi_{k,\omega}}{1+\varphi_{k,\omega}\left(\frac{1}{A_{\omega}}-1\right)+\varphi_{\omega,k}\left(\frac{1}{A_k}-1\right)};$$

b) If the perturbations are used to obtain the accomodation coefficient α,

$$(LT)_k=\frac{PF_k\beta}{\sqrt{B}}\sum_{\omega=1}^{l}\alpha_{\omega}T_{\omega}^{\frac{1}{2}}\varphi_{\omega,k}-\frac{PF_k\beta}{\sqrt{B}}\alpha_kT_k^{\frac{1}{2}}\sum_{\omega=1}^{l}\varphi_{\omega,k}.$$

If we consider the thermal regime of bodies exchanging heat with the surrounding medium by thermal conductivity,

$$(LT)_k=\lambda_k(\mathrm{grad}\,T)_k.$$

If the two instants of time 1 and 2 between which the perturbation is applied are so close together that the temperatures of the surfaces participating in the heat exchange do not change appreciably, the expressions for the difference of the operators LT simplify. We note that to determine the thermal constants of the surfaces or bodies it is not necessary to perturb exactly these thermal constants. The problem therefore arises as to which surfaces or bodies are the optimal for the thermal-constant perturbation in real situations.

NOTATION

A, A_k, and A_{ω}, the reflectivities of the bodies; α_k and α_{ω}, accommodation coefficients of the surfaces; B, molecular weight of the gas; β, coefficient depending on the number of degrees of freedom of the gas; C_{ef}, the effective value of the specific heat; ε, ε_k, and ε_{ω}, emissivities of the bodies; F_k and F_{ω}, surface areas of the bodies; $\varphi_{\omega,k}$ and $\varphi_{k,\omega}$, mutual irradiance coefficients; Kn, the Knudsen number; L, an operator; λ_k, heat conductivity; $\mu_{k,j,i}$, an arbitrary parameter; P, pressure; Q_k, and Q_ω, the rate of heat flow through the surfaces; Q_b, the flow rate of heat released inside the bodies; q_k, and $q_{k,j}$, the total heat flux and its components; σ, Stefan—Boltzmann constant; T, T_k, and T_ω, temperatures; τ, time.

STATISTICAL PARAMETER CORRECTION FOR MATHEMATICAL MODELS OF HEAT-ENGINEERING SYSTEMS

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The use of the Kalman-filter equations to calculate parameter corrections for mathematical models of heat-engineering systems is considered.

Recently, discrete (point) models have been used increasingly widely for the calculation and analysis of complex heat-engineering systems. However, the results of such calculations often disagree with experimental data. The sources of possible error may be conveniently divided into three groups [2]: incorrect determination of the functional (structural) design of the system, measurement errors, and errors in the choice of the model parameters.

In the first case, it is necessary to develop a new model. In the last two cases, it is possible to make a statistical estimate of the model parameters using the results of measurements, and so obtain corrected values.

Among the statistical methods used to estimate the parameters of heat-engineering-system models are algorithms based on the equations of the linear optimal Kalman filter; these are of recurrent form and allow the order of the matrices used in the calculations to be considerably reduced. In [1, 4, 5] the filter equations were used in the nonlinear problem of joint estimation of the paremeters and state by linearization of the initial equations in the vicinity of a preliminary estimate. In [2], an estimation problem with initial equations that were linear with respect to the parameters was considered, in the case when the accurate value of the state vector is known. In this formulation, the estimation problem becomes linear and direct solution is possible using the Kalman-filter equations [3]; essentially, it reduces to a recurrent least-squares method.

In the present work, the Kalman-filter equations are used in a parameter-estimation problem for a point model of a heat-engineering system, described by the difference matrix equation

$$\bar{t}(k+1) = A\bar{t}(k) + C\bar{q}(k), \tag{1}$$

or for an individual element

$$t_i(k+1) = t_i(k) + \sum_{j=i} \frac{\alpha_{ij}}{c_i}(t_j(k) - t_i(k)) + \frac{q_i(k)}{c_i}. \tag{2}$$

It is assumed that the value

$$\bar{t}^*(k) = \bar{t}(k) + \bar{n}_t(k) \tag{3}$$

is measured, and likewise for

$$\bar{q}^*(k) = \bar{q}(k) + \bar{n}_q(k), \tag{4}$$

where $\bar{n}_t(k)$ and $\bar{n}_q(k)$ are independent random Gaussian series of white-noise type with zero mean and covariance matrices $\mathrm{cov}(\bar{n}_t) = P$, $\mathrm{cov}(\bar{n}_q) = N$. The parameters $1/c_i$ and α_{ij}/c_i are to be estimated. Then, by identity transformations, the equations of state and of observation — Eqs. (1) and (3), respectively — may be reduced to the form

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